

stochastic: sec 5

⇒ Discrete Random Variable

The complete description of a discrete random variable requires specifying the possible values that the random variable may take and the probability associated with each value (probability distribution)

X	1	2	3	4	5	6
P(X)						

Mean - (expectation or expected value)

$$E(X) \text{ or } \mu_x \text{ or } \mu$$

$E(X) \rightarrow$ first moment of the origin

$$= \sum_x x P(x)$$

2nd moment of origin = $\sum_x X^2 P(x)$

rth " " " " = $\sum_x X^r P(x)$

* Variance

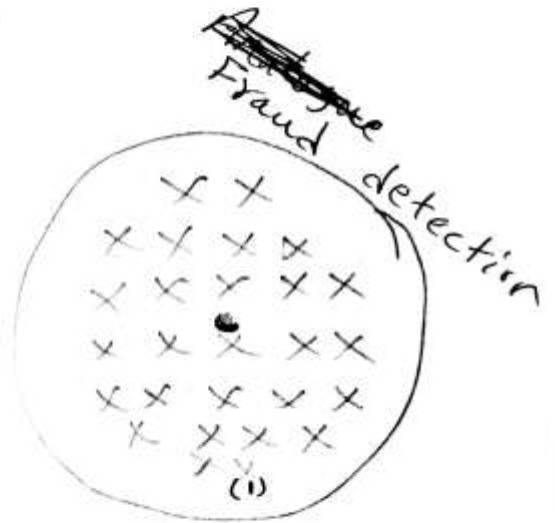
من دي مجموعة من ال (Samples)

لو جينا ال (mean) عند النقطة

في المتوسط "•" من لو عايز نجيب

ال (Variance) عند (1) هو مربع

المسافة بين النقطة وال (mean)



من مثال لو عندي بنك مثلاً فيه صلايين العملاء (Fraud detection)

فأحنا هنا هنعمل (relationship) بين العملاء لو كان

الرسم لو فيه (transactions) خرجوا من المجموعة

دي يكون فيه حد بيعمل ~~شيء~~ غسيل أموال.

~~فنتبع الشخص في~~

له الكلام ده فنتبعه فقط.

1st moment about the mean $= E(x - \mu)$

$$= \sum_{\mu} (x - \mu) P(x)$$

2nd moment about the mean $= \boxed{E(x - \mu)^2}$

$\downarrow$
Variance
 σ^2

$$= \sum_x (x - \mu)^2 P(x)$$

rth moment about the mean $= E(x - \mu)^r$

$$= \sum_x (x - \mu)^r P(x)$$

Standard Deviation σ (S.D)

$$\sigma = \sqrt{\text{Var}(x)}$$

[Ex] Prove that $\text{Var}(x) = E(x^2) - \mu^2$

Sol

$$\text{Var}(x) = \sum_x (x - \mu)^2 P(x)$$

[3]

$$= \sum_x (x^2 - 2\mu x + \mu^2) P(x)$$

$$= \sum_x x^2 P(x) - 2\mu \underbrace{\sum_x x P(x)}_{\mu} + \mu^2 \underbrace{\sum_x P(x)}_1$$

$$= \sum_x x^2 P(x) - 2\mu^2 + \mu^2$$

$$= E(x^2) - \mu^2 \neq$$

$\Rightarrow$ For Continuous Random Variable

* Mean

1st moment about the origin $= E(x)$

$$= \int_{-\infty}^{\infty} x P(x) dx$$

2nd ... $= E(x^2) = \int_{-\infty}^{\infty} x^2 P(x) dx$

Variance

1st moment about the mean $= E(X - \mu)$

$$= \int_{-\infty}^{\infty} (x - \mu) p(x) dx = 0$$

2nd $= E(X - \mu)^2 = \int_{-\infty}^{\infty} (x - \mu)^2 p(x) dx$

sheet 4

7 Plot the graph of $F(x)$ for the variable x with density.

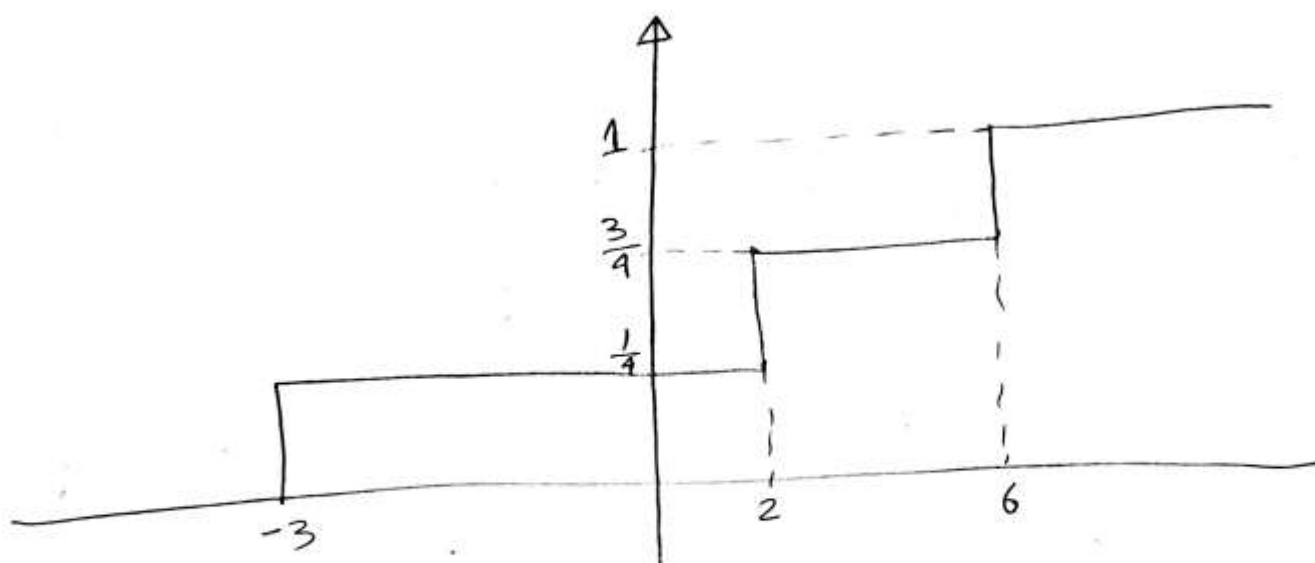
x	-3	2	6	
$p(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	

$$F(-\infty) = 0$$

$$F(-3) = P(-3) = \frac{1}{4}$$

$$F(2) = P(2) + P(-3) = \frac{3}{4}$$

$$F(6) = \cancel{P(6) + P(2) + P(-3)} = 1$$



Sheet 5

① X equal the no. of successful cures out of the five.

X	0	1	2	3	4	5
$P(X)$	0.002	0.029	0.132	0.309	0.36	0.168

② μ

③ σ^2

$$\mu = \sum x P(x) = 0(0.002) + 1(0.029) + 2(0.132) + 3(0.309) + 4(0.36) + 5(0.168) = 3.5$$

$$\sigma^2 = \sum (x - \mu)^2 P(x) = (0 - 3.5)^2 0.002 + (1 - 3.5)^2 0.029 + (2 - 3.5)^2 0.132 + (3 - 3.5)^2 0.309 + (4 - 3.5)^2 0.36 + (5 - 3.5)^2 0.168 = \dots$$

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$$\sigma = \sqrt{\sigma^2} = 1.02$$

[2] Find the expectation, Variance, ~~Standard~~ S.D.

iv)

$$P(x) = \begin{cases} \frac{2}{25} x & 0 \leq x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

$$\mu = \int_{-\infty}^{\infty} x P(x) dx = \int_0^5 \frac{2}{25} x^2 dx = \frac{2}{25} \frac{x^3}{3} \Big|_0^5$$

$$\mu = \frac{10}{3}$$

$$\text{Var} = E(x^2) - \mu^2$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 P(x) dx = 12.5$$

$$\text{Variance} = 12.5 - \left(\frac{10}{3}\right)^2 =$$

$$\sigma = \sqrt{\text{Var}} =$$

[7]